Published by Institute of Physics Publishing for SISSA/ISAS
Received: *September 22, 2003*
Accepted: *November 18, 2003*

On brane-induced gravity in warped backgrounds

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Abstract: We study whether modification of gravity at large distances is possible in warped backgrounds with two branes and a brane-induced term localized on one of the branes. We find that there are three large regions in the parameter space where the theory is weakly coupled up to high energies. In one of these regions gravity on the brane is four-dimensional at arbitrarily large distances, and the induced Einstein term results merely in the renormalization of the 4d Planck mass. In the other two regions the behavior of gravity changes at ultra-large distances; however, radion becomes a ghost. In parts of these regions, both branes have positive tensions, so the only reason for the appearance of the ghost field is the brane-induced term. In between these three regions, there are domains in the parameter space where gravity is strongly coupled at phenomenologically unacceptable low energy scale.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions.

Contents

1. Introduction

Emerging evidence for the accelerated expansion of the Universe triggered interest in the non-standard theories of gravity in which gravitational interactions get modified at length scales larger than a certain critical value r_c, which is assumed to be of the order of the present Hubble length. The motivation is that the unconventional behavior of gravity at cosmologically large length scales may either screen the effect of the huge vacuum energy or even be at the origin of cosmic acceleration if the vacuum energy is exactly zero.

Naively, the most straightforward way to modify gravity at the distance scale r_c is to give graviton a mass $m_g \sim r_c^{-1}$. However, it is known for a long time [1] that the massive gravity does not reproduce the conventional Einstein gravity at the linearized level in the limit $m_g \to 0$ due to the different tensor structures of the propagators for the massive and massless gravitons. It was argued [2] that this problem may be cured by taking into account non-linear effects. However, recent study [3] demonstrated that any four-dimensional theory of a single massive graviton either exhibits ghost fields or loses its predictive power in the UV due to strong quantum effects at the scale $\Lambda = \left(M_{Pl}/r_c^2\right)^{1/3}$. If r_c is of the order of the Hubble distance $H^{-1} \sim 10^{28}$ cm then $\Lambda \sim 10^{-8}$ cm^{-1} which is unacceptably low from the phenomenological point of view. This strong interaction is due to the fact that when the graviton mass term is added, extra scalar and vector degrees of freedom become dynamical and strongly interacting, and as a result there is no smooth limit $m_g \to 0$ (a notable exception from this rule is massive gravity in the adS space [4]).

This situation is somewhat reminiscent of what happens with non-Abelian Yang-Mills theory after gauge invariance is explicitly broken by the mass term, so one is tempted to try to find an analogue of the Higgs mechanism providing a predictive UV completion of massive gravity.

Models with infinite extra dimensions ("brane worlds") appear to be a natural framework for modifying gravity at large distance scales. Indeed, in brane world models four-dimensional physics is reproduced due to the presence of KK modes with wave-functions localized near our brane. A number of models [5]–[10] have been proposed where four-dimensional graviton is actually a quasilocalized state with a tiny but finite width Γ (and, possibly, mass). In these models one may expect that gravity is four-dimensional at distances shorter than $r_c \sim \min\{m^{-1}, \Gamma^{-1}\}$ and multi-dimensional at longer scales. Also one may hope that the presence of a continuum of light bulk modes gives rise to non-locality, which is strong enough to get around the no-go result obtained in the four-dimensional theory.

Unfortunately, this is not the case as yet. In the five-dimensional model of refs. [5, 6], where graviton is quasilocalized due to the warped geometry of the bulk space, a ghost field was found [11]. Its presence leads to the correct tensor structure of the graviton propagator at intermediate scales but makes the whole theory inconsistent. An interpretation is that this ghost is due to the presence of the dynamical negative tension brane in the model that violates the weakest energy condition.

In another class of models [8]–[10] the bulk space is flat and the graviton is quasilocalized due to the presence of the four-dimensional Einstein term on the brane. It has been argued that this term may be induced by the loops of particles localized on the brane. Naively, one does not expect ghosts in this setup as the positive energy condition is not violated. Surprisingly, tachyonic ghost field was found [12] in the model with the number of extra dimensions N greater than one. In fact, $N > 1$ models are somewhat special, because they exhibit singularity in the thin brane limit and need some regularization to resolve this singularity. In ref. [12] a particular regularization suggested in ref. [10] was used. One may consider alternative regularizations (see, e.g. refs. [13, 14]), where ghost fields are absent; however, then one again faces strong coupling at the unacceptably low energy scale.

The existence of ghosts in a setup where the positive energy condition is maintained may appear somewhat puzzling. To understand the situation, it is instructive to study the case of one extra dimension which is free of singularities. The flat space model [8] with induced Einstein term on the brane was studied in much detail in refs. [15, 16]. This model does not possess ghosts; however there is strong coupling like in the four-dimensional theory with massive graviton. The strongly interacting mode is basically a brane bending mode, so one may suspect that strong coupling is related to the fact that the brane has zero tension. Thus one is naturally led to consider models with non-zero brane tension and, in static situation, warped bulk space. The study of models with warped bulk space and induced term on the brane was started in ref. [15] (for earlier work, where the case of a scalar field was considered, see, e.g., ref. [19]). There the brane was assumed to be at the fixed point of the $\mathbb{Z}_2$ orbifold. In particular, it was shown that in the second Randall–Sundrum model with a single positive tension brane, the induced term does not change physics in the regime of validity of the four-dimensional effective theory and merely renormalizes the value of the Plank mass on the brane. However, gravity is not modified at large distances in that model.

In this paper we continue the study of models with warped bulk and induced Einstein term on the brane. We drop the assumption of the $\mathbb{Z}_2$ symmetry across our brane and consider the effect of the Einstein term on one of the branes in the models of the Lykken–

Randall type [17]. We demonstrate that the behavior of these models is in some respect similar to the behavior of massive four-dimensional gravity. Namely, there are regions in the parameter space where the theory is weakly coupled up to high energies. In one of these regions the induced term is localized on the Planck brane. In this region the induced Einstein term results merely in the renormalization of the 4d Planck mass (and the absence of 5d behavior in the UV), so that gravity is not modified at large distances. In the other regions the behavior of gravity changes at ultra-large distances; however, radion becomes a ghost. Different regions in the parameter space are separated by the domains where gravity is strongly coupled at unacceptably low energy scale. It is worth noting that there is a region in the parameter space where all branes have positive tensions, but the ghost field is still present in the weak coupling regime. This ghost is a radion, in a complete analogy with what happens in the GRS model [11]. The difference is that the GRS radion is a ghost because the tension of the TeV brane is negative, while in our case it is the presence of the induced term that makes the radion to be a ghost.

The structure of this paper is as follows. In section 2 we describe the setup. In section 3 we present a simple technique, based on the study of the junction conditions on the brane, which enables one to single out the strong coupling domains in the parameter space. We demonstrate that in the model under consideration there are two strong coupling domains separating three weak coupling regions. In one of the latter regions the induced term is localized on the Planck brane and gravity is exactly localized. In the other two regions the induced term is on the TeV brane. Then one may expect modification of gravity at large distances. However, in section 4 we demonstrate that the radion is a ghost in these regions. We present our conclusions in section 5.

2. The model

We consider the five-dimensional background with the metric

$$ds^2 = a^2(z)\eta_{\mu\nu}dx^\mu dx^\nu - dz^2 \,. \tag{2.1}$$

We assume that this metric is a solution of the five-dimensional gravity with three-brane sources,

$$S = -M^3\int d^5X\sqrt{g}R^{(5)} - \Lambda\int d^5x\sqrt{g} - \sum_i \lambda_i \int d^5X\sqrt{-\gamma^{(i)}}\delta(z-z_i)\,, \tag{2.2}$$

where g is the bulk metric and $\gamma^{(i)}$ is the metric induced on the i-th brane. The five-dimensional cosmological constant $\Lambda < 0$ may be different in different domains of the bulk space, separated by three-branes. We consider the Lykken–Randall model with two branes. The first ("hidden") brane is placed at the fixed point $z = -z_h$ of the $\mathbb{Z}_2$ orbifold symmetry $z \to -2z_h - z$. The z coordinate of the second ("visible") brane is $z = 0$. Also we add to the action the induced Einstein term

$$S_{\text{ind}} = -M^2_{\text{ind}}\int d^5X\sqrt{-\gamma^{(v)}}\delta(z)R^{(4)}(\gamma^{(v)}) \tag{2.3}$$

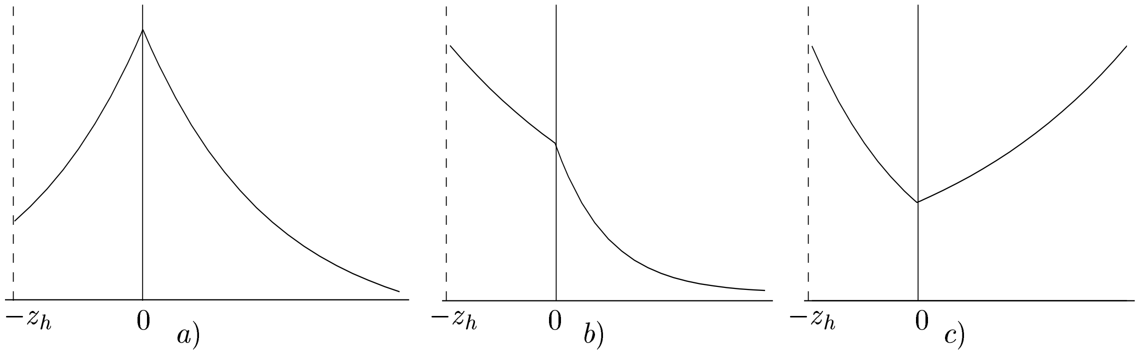

Figure 1: Profiles of the warp factor $a(z)$ for different choices of signs for k_L and k_R: $a)$ $k_L < 0$, $k_R > 0$ $b)$ $k_L > 0$, $k_R > 0$ $c)$ $k_L > 0$, $k_R < 0$. Solid and dashed vertical lines show the position of the visible brane and the hidden brane, respectively.

localized on the visible brane. The explicit shape of the warp factor $a(z)$ is

$$a(z) = \begin{cases} \mathrm{e}^{-k_L z}\,, & -z_h < z < 0 \\ \mathrm{e}^{-k_R z}\,, & 0 < z\,, \end{cases} \tag{2.4}$$

where the values of the inverse adS radius on the left and on the right of the visible brane $k_{L,R}$ are related to the values of the cosmological constant on the left and on the right of the visible brane $\Lambda_{L,R}$ as follows,

$$|k_{L,R}| = \sqrt{-\frac{\Lambda_{L,R}}{12M^3}}\,. \tag{2.5}$$

The brane tensions should satisfy the fine-tuning conditions

$$\lambda_h = 12M^3 k_L\,, \qquad \lambda_v = 6(k_R - k_L)M^3\,. \tag{2.6}$$

The profile of the warp factor $a(z)$ for different choices of sign in eq. (2.5) is shown in figure 1. In what follows we do not consider the case $k_R < 0$ in detail. In this case one would have to introduce the third brane to screen the adS boundary at $z = +\infty$; as we will comment later on, we do not expect any qualitatively new features in that case.

Let us briefly recall the main properties of the two-brane model without induced Einstein term for different values of the parameters k_L and k_R.

If $k_L < 0$ and $k_R > 0$ one can push the hidden brane to infinity. Then at $k_L = -k_R$ one has just the second Randall-Sundrum model, while in general one arrives at the $\mathbb{Z}_2$-asymmetric generalization of that model. Due to the presence of a graviton zero mode localized on the visible brane, one recovers the conventional four dimensional gravity at large distances along this brane. The value of the four-dimensional Planck mass is

$$M_0^2 = M^3\left(\frac{1}{k_R} - \frac{1}{k_L}\right)\,. \tag{2.7}$$

At distances shorter than $r_b = \max\{k_L^{-1}, -k_R^{-1}\}$ bulk corrections are essential, while quantum gravity effects are important at even shorter distances of order M^{-1}.

If both k_L and k_R are positive then one cannot push the hidden brane to infinity. This brane is needed to screen the adS boundary at $z = -\infty$. In this case the wave function of the graviton zero mode is peaked near the hidden brane, so the effective Planck mass on the visible brane depends exponentially on the separation between the branes,

$$M_0^2 = M^3 \left(\frac{1}{k_R} + \frac{1}{k_L} \left(\mathrm{e}^{2z_v k_L} - 1 \right) \right) . \tag{2.8}$$

The long-distance potential $V(r)$ between two masses m_1 and m_2 on the visible brane at the distance r is given by[1][17, 18]

$$V(r) = \frac{1}{M_0^2} \frac{1}{r} \left(1 + \frac{M^3}{M_0^2 k^3 r^2} + \frac{M_0^2}{k^5 M^3 r^6} \right) . \tag{2.9}$$

Consequently, the four-dimensional Newton's law does not hold at distances shorter than

$$r_b = \frac{M_0^{1/3}}{k^{5/6} M^{1/2}} .$$

Note, that this distance becomes larger when the distance between the branes grows.

One more difference with the first case is that one does not recover the standard Einstein gravity on the visible brane at long distances. Instead, one has a scalar-tensor theory of gravity due to the presence of the radion field.

Another feature of this model is that if $k_R < k_L$, then the tension of the visible brane is negative (see eq. (2.6)) and the radion becomes a ghost [11]. For $k_R = 0$ one recovers the GRS model with quasilocalized gravity and ghost field.

The purpose of this paper is to study the modification of the above results due to the presence of the induced term (2.3) and in particular to check that there is no corner of the parameter space with long distance modification of gravity and without ghosts or strong coupling at unacceptably low energy scale.

For the moment let us ignore the fact that the graviton is a tensor particle and briefly discuss the effect one would expect from the induced term in the scalar case (more detailed discussion of the scalar propagator in the Randall-Sundrum model with induced term can be found in ref. [19]). The brane-to-brane scalar propagator $G(p)$ in the presence of the induced term is

$$G(p) = \frac{1}{M^3 \left(G_0^{-1} + r_c p^2 \right)} , \tag{2.10}$$

where $G_0(p)$ is the brane-to-brane propagator without induced term and

$$r_c = \frac{M_{\text{ind}}^2}{M^3} .$$

At distances shorter than the adS radius one has $G_0(p) = 1/p$, so that at short distances the propagator $G(p)$ always has a four-dimensional form with the Planck mass equal to M_{ind}.

[1]Strictly speaking, this result was obtained in the case of the zero tension brane $k_L = k_R$. Corrections to eq. (2.9) present at $k_L \neq k_R \equiv k$ do not affect our discussion here.

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At very large distances one again recovers four-dimensional propagator with Planck mass

$$M_l^2 = M_{\rm ind}^2 + M_0^2\,. \tag{2.11}$$

So in principle one can have modification of gravity at long distances in this model. Namely, at relatively "short" distances one expects 4d gravity with Planck mass $M_{\rm ind}$, then a region of scales where bulk contribution dominates, and 4d gravity with Planck mass M_l at longer scales. Arranging the parameters in such a way that the first of these transitions occurs at the cosmological scale, one would hope to obtain interesting long-distance modification of gravity. Furthermore, for positive k_L and large enough distance z_h between the branes, the second of these transitions occurs at the length scale longer than the curvature length in the bulk. Unfortunately, as we will see in the rest of the paper, whenever large distance modification of gravity takes place, there is either ghost or strong coupling at unacceptably low energy scale.

3. Junction conditions and strong coupling

In this paper we study the effect of the induced term (2.3) in the regime when the parameter $M_{\rm ind}$ is the largest energy scale involved in the problem,

$$M_{\rm ind} \gg k_L\,,\ k_R\,,\ M\,.$$

In flat space model with brane-induced gravity the following peculiar effect happens [15, 16]. Some couplings determining the interaction strength of the longitudinal (from 4d point of view) components of the graviton involve positive powers of the parameter $M_{\rm ind}$. As a result, a new dynamical scale

$$\Lambda = \frac{M^2}{M_{\rm ind}}$$

emerges. Above this energy scale the theory is strongly coupled. In what follows, we refer to this situation as strong coupling regime. It is shown in ref. [15], that, unlike the case of flat bulk, in the second Randall-Sundrum model with the induced term the strong coupling regime does not occur if

$$M_{\rm ind}^2 > \frac{M^3}{k}\,.$$

In this section we describe a simple technique to single out regions of the parameter space where strong coupling regime takes place. A straightforward way [16] to see the strong coupling is to calculate the full propagator in the theory. If this propagator contains "large" terms proportional to positive powers of $M_{\rm ind}$ which are not pure gauge everywhere, then strong coupling regime takes place. A shortcut which we make use of here is the observation that these terms show up already in the junction conditions for the metric perturbations on the visible brane. The first junction condition we employ is the Israel condition [20] for the discontinuity of the extrinsic curvature $K_{\mu\nu}$. In the presence of the induced term this condition takes the following form

$$[K_{\mu\nu}]_-^+ = \frac{1}{M^3}\left(-\frac{1}{6}\lambda\gamma_{\mu\nu}^{(v)} + \frac{1}{2}\left(\tau_{\mu\nu} - \frac{1}{3}\gamma_{\mu\nu}^{(v)}\tau\right) - M_{\rm ind}^2\left(R_{\mu\nu}^{\rm ind} - \frac{1}{6}\gamma_{\mu\nu}^{(v)}R^{\rm ind}\right)\right)\,, \tag{3.1}$$

where $\tau_{\mu\nu}$ is the energy-momentum tensor of matter residing on the brane ($\tau_{\mu\nu}$ does not include the brane tension).

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To obtain the second equation, let us present the projection of the Einstein tensor on direction normal to the brane in the following form[2]

$$G_{AB}n^A n^B = R_{ABCD}\left(g^{AC}+n^A n^C\right)\left(g^{BD}+n^B n^D\right)\,, \tag{3.2}$$

where R_{ABCD} is the bulk Riemann tensor and n^A is the unit vector normal to the brane. Now, using the Gauss-Codacci relation, one may express the quantity in the r.h.s. of eq. (3.2) through the induced and extrinsic curvatures as follows

$$R_{ABCD}\left(g^{AC}+n^A n^C\right)\left(g^{BD}+n^B n^D\right) = R^{\mathrm{ind}} + \left(K^\mu_\mu\right)^2 - K_{\mu\nu}K^{\mu\nu}\,. \tag{3.3}$$

Finally, taking the component of the Einstein equations normal to the brane and using eqs. (3.2), (3.3) , one gets

$$(K^\mu_\mu)^2 - K_{\mu\nu}K^{\mu\nu} + \frac{\Lambda}{M^3} - \frac{T_{AB}n^A n^B}{M^3} + R^{\mathrm{ind}} = 0\,, \tag{3.4}$$

where the T_{AB} is the energy-momentum tensor of the bulk matter. It is worth pointing out, that in the initial value formulation of general relativity as a constrained Hamiltonian system, with the induced metric $\gamma_{\mu\nu}$ and extrinsic curvature $K_{\mu\nu}$ being the dynamical coordinates and momenta, eq. (3.4) plays the role of the constraint on their initial values (see, e.g., ref. [21]). It is worth noting that the constraint (3.4) is valid on both sides of the brane separately, so it actually provides *two* equations.

The strong coupling phenomenon happens in the scalar sector, so let us consider the trace of the Israel condition (3.1). Also, let us decompose the extrinsic curvature as follows

$$K_{\mu\nu} = \frac{a'}{a}\gamma^{(v)}_{\mu\nu} + \kappa_{\mu\nu}\,.$$

For the background metric (2.1) one has $\kappa_{\mu\nu} = 0$, $\gamma^{(v)}_{\mu\nu} = \eta_{\mu\nu}$ and the junction conditions (3.1), (3.4) imply the fine-tuning relations (2.5), (2.6). To the leading order in the deviations from the background, one has from eqs. (3.1), (3.4)

$$\begin{aligned}
[\kappa]^+_- &= -\frac{1}{M^3}\left(\frac{1}{6}\tau + \frac{1}{3}M^2_{\mathrm{ind}}R^{(\mathrm{ind})}\right)\\
-6\frac{a'}{a}\kappa + \frac{T_{zz}}{M^3} &= R^{(\mathrm{ind})}\,,
\end{aligned} \tag{3.5}$$

where $\kappa \equiv \eta^{\mu\nu}\kappa_{\mu\nu}$. Again, we actually have three equations, and using them one can find the values of $\kappa_{R(L)}$ on both sides of the brane and the induced scalar curvature $R^{(ind)}$. The result is

$$R^{(\mathrm{ind})} = \frac{1}{(k_R-k_L)M^3 - 2k_R k_L M^2_{\mathrm{ind}}}\left((k_R-k_L)T_{zz} + \frac{1}{2}\tau\right) \tag{3.6}$$

$$\kappa_{R(L)} = \frac{k_{L(R)}}{(k_R-k_L)M^3 - 2k_R k_L M^2_{\mathrm{ind}}}\left(\frac{M^2_{\mathrm{ind}}}{3M^3}T_{zz} + \frac{1}{6}\tau\right) \tag{3.7}$$

[2]Here Latin indices A, B take values μ, z, while Greek indices μ, ν correspond to the coordinates tangent to the brane.

eq. (3.6) implies that the induced curvature scalar never contains large contributions proportional to the positive powers of $M_{\rm ind}$. This is the reflection of the fact that large terms in the full propagator are pure gauge from the 4d point of view (cf. refs. [15, 16]). On the other hand, there is a potentially large term proportional to T_{zz} in the expression for the trace of the extrinsic curvature. If

$$r_c = \frac{M_{\rm ind}^2}{M^3} \gg \left| \frac{1}{k_L} - \frac{1}{k_R} \right| \tag{3.8}$$

the factor of $M_{\rm ind}^2$ in this term cancels out and there is no new strong coupling scale suppressed by $M_{\rm ind}^{-1}$. However, in the opposite case, the expression for the extrinsic curvature κ contains a term proportional to $M_{\rm ind}^2$; consequently, a term enhanced by $M_{\rm ind}^2$ is present in the Green's function and the strong coupling regime occurs.

In figure 2 the plane of the parameters k_L and k_R is shown. In the shaded grey region one has

$$r_c \lesssim \left| \frac{1}{k_L} - \frac{1}{k_R} \right|$$

so that strong coupling phenomenon happens there. The physical meaning of this region is clear: here the distance r_c is smaller then the adS radius on one of the sides of the visible brane. Consequently, the strong coupling is nothing but the same phenomenon as observed in the flat model. One can estimate the scale Λ of the strong coupling in the following way. Let us, for definiteness, consider the case $k_R > k_L$. From eq. (3.7) one finds that on the left of the visible brane one has

$$\kappa_L \sim \frac{M_{\rm ind}^2}{M^6} T_{zz} \, .$$

Consequently, the graviton propagator contains a large term proportional to $M_{\rm ind}^2/M^6$. Now, the three-graviton vertex becomes large if

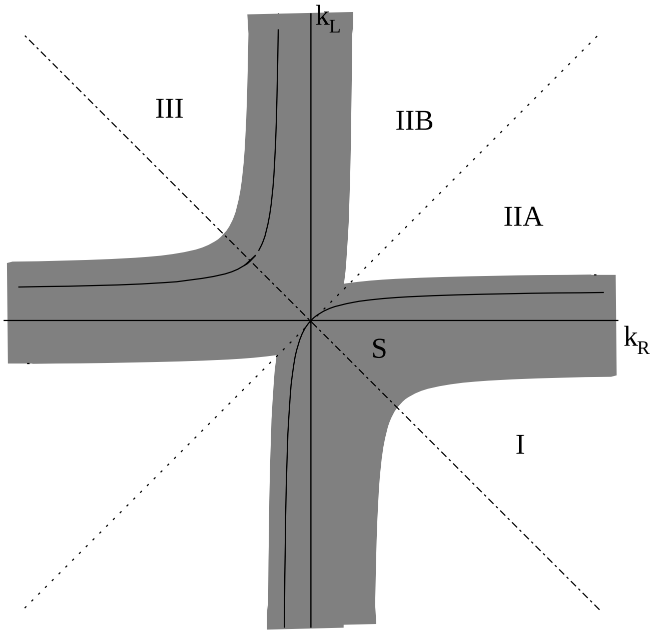

Figure 2: Parameter space of the two brane model with induced term. Strong coupling occurs in the grey shaded region S. Solid lines in this region are lines of zeroes of the denominator in eq. (3.7). Below the dotted line both branes have positive tensions. On the dash-dotted line one can perform $\mathbb{Z}_2$ identification on the visible brane and arrive at the Randall-Sundrum model.

$$M^3 \left(\frac{M_{\rm ind}^2}{M^6} \right)^{3/2} E^3 > 1 \, .$$

where a factor of M^3 comes from the vertex[3] and the power of energy E is restored on

[3]Note that, as we discussed above, the large term in the graviton propagator is pure gauge from the 4d point of view, so it cancels in the vertices coming from the brane-induced action.

dimensional grounds. This gives

$$\Lambda \sim \frac{M^2}{M_{\rm ind}}$$

in agreement with the analysis in the flat case.

In the region I in figure 2 one has $k_L < 0$, the induced term is on the Planck brane so one can remove the hidden brane. From the comparison of eqs. (3.8), (2.11) and (2.7) it follows that the absence of the strong coupling implies here that the Planck masses in the UV and IR are both approximately equal to $M_{\rm ind}$, so no large scale modification of gravity happens. Note, however, that the quantum gravity scale Λ is changed as compared to the case $M_{\rm ind} = 0$ (when it is equal to M) even in this region. Let us consider, for example, the $\mathbb{Z}_2$ symmetric case $|k_L| = k_R = k$. Then it follows from eq. (3.7) that the graviton propagator contains a term proportional to $1/(M^3 k)$ and the same analysis as above implies that

$$\Lambda \sim \sqrt{Mk}$$

in agreement with the result in ref. [15] for the Randall-Sundrum model with a single brane.

The analysis of the region I remains valid if the hidden brane is at finite distance z_h. Again, the 4d Planck mass is approximately equal to $M_{\rm ind}$, there is no long distance modification of gravity, but the strong coupling scale is below the 4d Planck mass and fundamental mass M.

As we discussed above, in the region IIA in figure 2 both branes have positive tensions and, at large enough separation between the branes, one may expect long distance modification of gravity. However, as we show below, radion is a ghost in the whole region II.

4. Effective action for the radion

In this section we consider the region $k_L > 0$. We calculate the quadratic effective action for the radion field and demonstrate that the radion is a ghost if

$$M_{\rm ind}^2 > \frac{M^3}{2}\left(\frac{1}{k_L} - \frac{1}{k_R}\right) \tag{4.1}$$

i.e., above the right solid line in figure 2. The calculation of the quadratic effective action is very similar to that in the case of the Lykken-Randall model without induced term [11]. First, following ref. [5], we calculate the wave function of the radion in two different patches, corresponding to the Gaussian normal coordinates with respect to the hidden and visible branes, so that the metric has the form

$$ds^2 = a^2(z)(\eta_{\mu\nu} + h^{(i)}_{\mu\nu})dx^\mu dx^\nu - dz^2\,,$$

where $i = \alpha, \beta$ labels different patches. Then we perform a gauge transformation to a single patch and plug the resulting wave function into the quadratic action.

The calculation of the radion wave function is completely parallel to that in ref. [5], so we just present the results. In the first patch, that covers the region $-z_h \le z < 0$, one has

$$h^{(\alpha)}_{\mu\nu} = f\eta_{\mu\nu} - \partial_\mu\partial_\nu f\frac{\mathrm{e}^{2k_L z}}{2k_L^2} + \partial_\mu\partial_\nu f\frac{\mathrm{e}^{2k_L(2z+z_h)}}{4k_L^2}\,. \tag{4.2}$$

Here $f(x)$ is the (unnormalized) radion field. The on-shell condition for the radion is $\partial_\mu \partial^\mu f = 0$. In the second patch, that covers the region $-z_h < z < \infty$ one has

$$h_{\mu\nu}^{(\beta)} = \begin{cases} C\left(f\eta_{\mu\nu} - \partial_\mu\partial_\nu f \dfrac{\mathrm{e}^{2k_L z)}}{2k_L^2}\right) + \partial_\mu\partial_\nu f \dfrac{\mathrm{e}^{2k_L(2z+z_h)}}{4k_L^2} & \text{for } z<0 \\ C\left(f\eta_{\mu\nu} - \partial_\mu\partial_\nu f \dfrac{\mathrm{e}^{2k_R z}}{2k_R^2}\right) + \left(\dfrac{C}{2}\left(\dfrac{1}{k_R^2} - \dfrac{1}{k_L^2}\right) + \dfrac{\mathrm{e}^{2k_L z_h}}{4k_L^2}\right)\partial_\mu\partial_\nu f & \text{for } z>0\,, \end{cases} \tag{4.3}$$

where

$$C = \frac{\mathrm{e}^{2k_L z_h} k_R}{k_R - k_L - 2M_{\text{ind}}^2 k_R k_L / M^3}\,,$$

$C < 0$ in the regions IIA and IIB in figure 2. Note, that the wave function (4.3) is pure gauge for $z > 0$, so the growth of $h_{\mu\nu}^{(\beta)}$ toward $z = +\infty$ is not dangerous. To calculate the effective action for the radion field let us perform gauge transformation to the coordinate system covering the whole space. This gauge transformation can be performed in two steps. First, one makes the shift of z-coordinate in the second patch

$$z \to z - \frac{(C-1)}{2k_L} f\,. \tag{4.4}$$

The resulting metric is described by a single patch, however, the visible brane is bended in this coordinate system. To eliminate this bending one performs the second gauge transformation

$$\begin{aligned} z &\to z + \frac{C-1}{2k_L} f B(z) \\ x^\mu &\to x^\mu + \xi^\mu(x,z)\,, \end{aligned} \tag{4.5}$$

where $B(z)$ is an arbitrary function satisfying

$$B(0) = 0\,, \qquad B(z_v) = 1$$

and functions $\xi^\mu(x,z)$ are fixed by the requirement that the (μz)-components of the resulting metric vanish. Explicitly one has

$$\xi^\mu = -a^2 \frac{C-1}{2k_L} B(z) \eta^{\mu\nu} \partial_\nu f\,.$$

The resulting metric has the form

$$ds^2 = a^2(z)(\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu - (1 - h_{zz}) dz^2\,,$$

where

$$h_{zz} = \frac{C-1}{2k_L} f B'(z) \tag{4.6}$$

$$h_{\mu\nu} = h_{\mu\nu}^{(\alpha)} - \frac{\mathrm{e}^{-2k_L(z-z_v)}(C-1)}{2k_L} \partial_\mu\partial_\nu f \int dz B(z) \mathrm{e}^{2k_L(z-z_v)} + 2(C-1)\eta_{\mu\nu} f B(z) \tag{4.7}$$

for $-z_h \leq z \leq 0$ and pure gauge for $z > 0$. To calculate the effective action for the radion we plug expressions (4.6) and (4.7) into the quadratic action presented in the form

$$S_2 = -\frac{1}{2}\int dz d^4x \sqrt{g}\delta g^{AB} E_{AB}[\delta g]$$

and take the integral over z-coordinate. Here tensor $E_{AB}[\delta g]$ is the linear part of the variation of the action given by eqs. (2.2) and (2.3). As in the case of the Lykken-Randall model without induced term [11], only (zz)-components of the tensor $E_{AB}[\delta g]$ are non-zero. Finally, one arrives at the following effective action for the radion

$$\begin{aligned} S_{\rm rad} &= -3\mathrm{e}^{-2k_L z_v} M^3 \frac{C-1}{2k_L}\int_0^{z_v} dz B'(z) \int d^4x f \partial_\mu^2 f \\ &= -\frac{3}{2}\mathrm{e}^{-2k_L z_v}\frac{M^3}{k_L}\left(\frac{\mathrm{e}^{2k_L z_v} k_R}{k_R - k_L - 2M_{\rm ind}^2 k_R k_L/M^3} - 1\right)\int d^4x f \partial_\mu^2 f \end{aligned} \qquad (4.8)$$

We see, that the radion is indeed a ghost in the regions IIA and IIB in figure 2.

5. Concluding remarks

To conclude, let us first comment on what happens in the region III which was not discussed so far. When k_R becomes negative, the warp factor $a(z)$ has the shape shown in figure 1c) and the third brane is needed to screen the adS boundary at $z = +\infty$. Consequently, the second radion appears here. This radion becomes a ghost above the left solid line in figure 2. Consequently, there are two ghost fields in the region III. On the dash-dotted line one can make $\mathbb{Z}_2$ identification with respect to the visible brane. This identification removes one of the ghosts in the region III, but the second one is still there in agreement with ref. [15].

To summarize, the results of this paper suggest that it is unlikely to cure the problems of the brane-induced gravity models by invoking branes of non-zero tension and warped bulk space. These results as well as those obtained in refs. [11, 12, 15, 16] indicate that non-locality in the low-energy effective theory arising due to the presence of infinitely large extra dimensions is not sufficient to get around the no-go results of ref. [3] and some more radical approach is needed to achieve large scale modification of gravity in the consistent theory.

Acknowledgments

We would like to thank Yu. Kubyshin, D. Levkov, M. Shaposhnikov, S. Sibiryakov, P. Tinyakov, and especially V. Rubakov and R. Rattazzi for helpful discussions. S.D. would like to thank the IPT at the Lausanne Univ., where part of this work was done, for kind hospitality.

This work is supported in part by the IISN (Belgium), the "Communauté Française de Belgique"(ARC), and the Belgium Federal Government (IUAP) (M.L.); by RFFI grant 02-02-17398; by the Grants of the President of the Russian Federation NS-2184.2003.2; by the programme SCOPES of the Swiss NSF, project No. 7SUPJ062239, financed by Federal Department of Foreign affairs.

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